

Analysis of the viewing zone of the Cambridge autostereoscopic display

N. A. Dodgson

The Cambridge autostereoscopic three-dimensional display is a time-multiplexed device that gives both stereo and movement parallax to the viewer without the need for any special glasses. This analysis derives the size and position of the fully illuminated, and hence useful, viewing zone for a Cambridge display. The viewing zone of such a display is shown to be completely determined by four parameters: the width of the screen, the optimal distance of the viewer from the screen, the width over which an image can be seen across the whole screen at this optimal distance, and the number of views. A display's viewing zone can thus be completely described without reference to the internal implementation of the device. An equation that describes what the eye sees from any position in front of the display is derived. The equations derived can be used in both the analysis and design of this type of time-multiplexed autostereoscopic display. © 1996 Optical Society of America

1. Introduction

Autostereoscopic displays offer the viewer three-dimensional realism that is lacking in conventional two-dimensional or stereoscopic displays. The combination of both stereo parallax and movement parallax produces a perceived effect similar to a white-light hologram.

In real life we gain three-dimensional information from a variety of cues. Two important cues are stereo parallax, in which we see a different image with each eye, and movement parallax, in which we see different images when we move our heads. Figure 1(a) shows an observer looking at a scene. The observer sees a different image of the scene with each eye and different images again whenever he or she moves his or her head. The observer is able to view a potentially infinite number of different images of the scene.

Figure 1(b) shows the same viewing space divided into a finite number of *windows*. In each window only one image, or *view*, of the scene is visible. However, the viewer's two eyes each see a different image, and the images change when the viewer moves his or her head—albeit with jumps as the

viewer moves from window to window. Thus both stereo and movement parallax cues can be provided with a small number of views.

The finite number of views required in Fig. 1(b) permits the replacement of the scene by a three-dimensional display that outputs a different image to each window [Fig. 1(c)]. This is the principle of multiview autostereoscopic displays.

A. Autostereoscopic Display Technologies

A variety of autostereoscopic technologies have been developed. Lenticular displays and hologram displays use high-resolution display devices to produce multiview images at a lower resolution. Lenticular displays¹ use subpixels beneath microlenses. They normally provide two views, which does not provide movement parallax. Both a four-view² and an eight-view³ lenticular display have been demonstrated, but precise alignment of microlenses and pixel array and the high resolution required make more than four views difficult to achieve.

Hologram displays use a pixellation fine enough to form diffraction gratings. There is potential for hundreds of views to be displayed.^{4,5} However, the resolution required to make a diffraction grating necessitates that the equipment be mounted on an optical bench and that a supercomputer be used to drive the display.

Parallax barriers¹ provide a more flexible two-view alternative to lenticular screens but suffer the same problems when one tries to increase the number of views. Multiple projector systems⁶ avoid the

This is a preprint of a paper that appears in *Applied Optics* 35(10):1705–1710, 1 April 1996. <https://doi.org/10.1364/AO.35.001705>

Neil Dodgson was with the University of Cambridge when this work was published. <http://www.neildodgson.com>

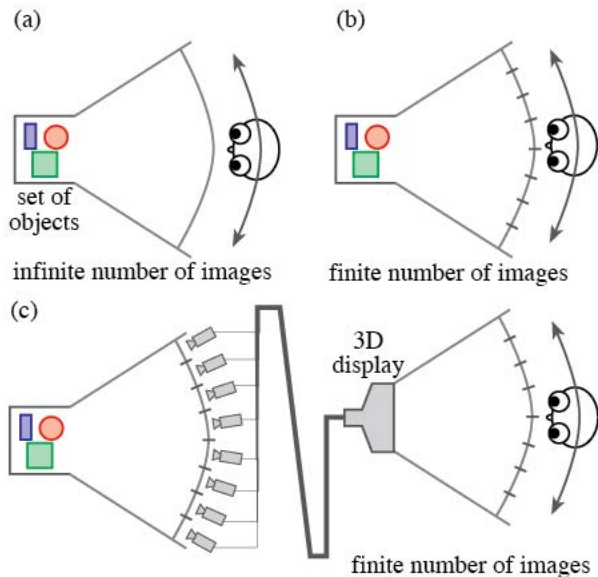


Fig. 1. (a) In viewing a real-world scene, there are an infinite number of possible images of the scene. (b) It is possible to divide this viewing space conceptually into a finite number of windows, in each of which only a single image is visible, while still retaining both stereo and movement parallax cues. (c) An autostereoscopic three-dimensional display uses this idea to provide a three-dimensional image, using a finite number of views taken from distinct view points.

resolution problem by using several projection devices that image through a double lenticular lens array. Although this undoubtedly works, it is expensive in that one projector is required per view, and it is difficult to align the projectors precisely for comfortable viewing.

All these methods provide multiple views by having more spatial resolution than an equivalent two-dimensional display. An alternative is to have a higher frame rate. A two-dimensional display is made visible to one window at a time and the appropriate image is displayed. If this process is repeated sufficiently rapidly, the whole seems continuous to the human eye. There are no misalignments between the views because all the views are displayed on the same device. This time-multiplexed method has the advantage that it is easier to increase frame rate than resolution.

The Cambridge autostereoscopic display⁷ uses such a temporally multiplexed system to achieve a laterally multiplexed autostereoscopic image. This paper derives equations that describe the behavior of an ideal Cambridge display.

B. Cambridge Display

The basic design of a Cambridge display [Fig. 2(a)] consists of a high-speed liquid-crystal display, a convex lens, and a series of abutting bar-shaped light sources. Each light bar is illuminated in turn. In synchronization with this, successive laterally adjacent views of an object are displayed on the liquid-

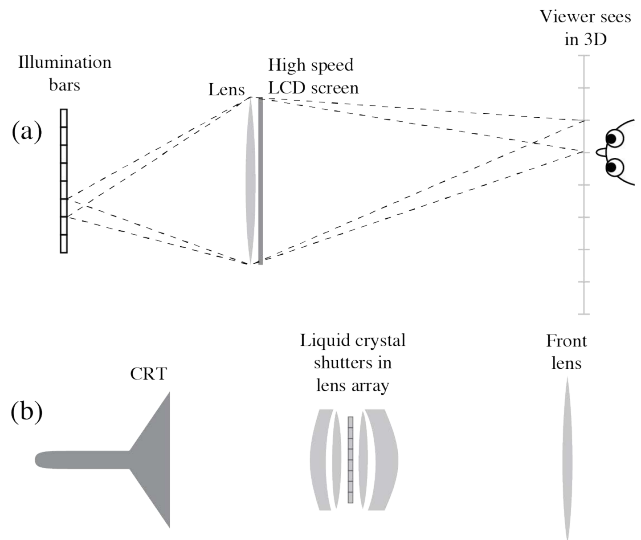


Fig. 2. (a) Conceptual design of a Cambridge autostereoscopic display. (b) Currently practicable design in operation at Cambridge. LCD, liquid-crystal display.

crystal display. The effect of the lens is that each view is visible from a different set of directions in front of the display. Provided that the views are repeatedly illuminated sufficiently rapidly, an observer will perceive a three-dimensional image.

Eight views displayed at a 60-Hz refresh rate requires a liquid-crystal display with a frame rate of 480 Hz. A more desirable 32 views would require almost 2 kHz. Neither speed is feasible with nematic liquid crystals but may be attainable with smectic liquid crystals if the problem of transferring image data sufficiently quickly to the liquid-crystal array can be overcome.⁸

A practicable 16-view version of a Cambridge display has been built at the University of Cambridge.⁹⁻¹¹ It utilizes a CRT with 1-kHz frame rate, a projection lens, and a smectic liquid-crystal display element [Fig. 2(b)]. It is capable of 16 views at 640×240 resolution or 8 views at 640×480 . The CRT version emulates the liquid-crystal display and illumination system of the basic design. It is functionally identical to the ideal design, and the following analysis is equally applicable to either.

The principle behind the display has been generally understood to be directional modulation, with the optical system ensuring that each of the views is visible over only a small range of directions, as illustrated in Fig. 2(a). The actual behavior of the display is considerably more complex than this simple description, and up to this time it has been understood only from qualitative observations of the display. The analysis presented here is the first quantitative description of the behavior of the Cambridge display. It successfully predicts and explains the effects of the existing CRT-based displays and has proved effective in preparing designs for future models.

2. Basic Parameters

The Cambridge display (Fig. 3) consists of a simple convex lens and an adjacent display screen of width w_l , and a set of N illumination bars with overall width w_b . The bars are situated a distance d_b from the lens. The lens has a focal length f .

The system is arranged such that $d_b \geq f$. Consequently, an image of the illumination bars is projected a distance d_o in front of the lens. This image has width w_o . These two parameters are related to d_b and w_b by the simple equations

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_b}, \quad (1)$$

$$\frac{w_b}{d_b} = \frac{w_o}{d_o}. \quad (2)$$

An eye at d_o will see the entire screen illuminated by a point on one of the illumination bars. Each of the viewer's two eyes will be illuminated by a different bar, and hence will see a different view. This provides stereo parallax to the viewer. When the observer moves his or her head left to right at distance d_o , the observer's eyes will move through zones illuminated by different bars. This provides movement parallax, allowing the observer to look around objects in the image. The combination of these two effects produces a powerful three-dimensional illusion.

Viewing at other distances still produces a three-dimensional illusion. The purpose of the following analysis is to ascertain what the viewer will see from any position in front of the screen. To achieve this we must find which parts of the illumination system illuminate the screen for all positions of the eye. This will allow us to quantify the zone over which a viewer will perceive a three-dimensional effect.

3. Pupil's Image on the Illumination Bars

If an eye is placed at an arbitrary point (z, x) in front of the screen (Fig. 4), the image of an idealized

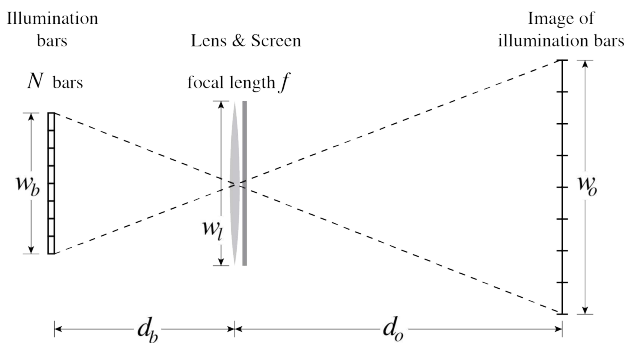


Fig. 3. Basic parameters of a Cambridge autostereoscopic display.

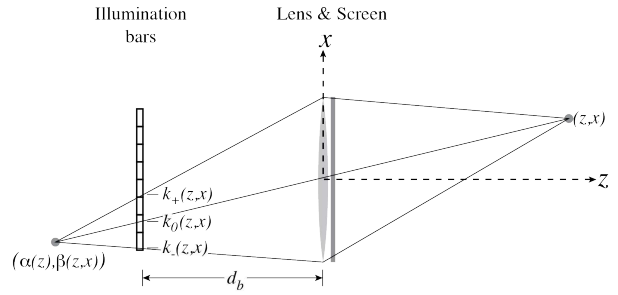


Fig. 4. Image of a pupil at (z, x) is $(\alpha(z), \beta(z, x))$. At distance d_b behind the lens, the point (z, x) images onto the area from $k_-(z, x)$ to $k_+(z, x)$.

pinhole pupil will be at $(\alpha(z), \beta(z, x))$, where

$$\alpha(z) = \frac{1}{(1/z) - (1/f)}, \quad (3)$$

$$\beta(z, x) = \frac{x}{1 - (z/f)}. \quad (4)$$

The area imaged by the point (z, x) at distance d_b behind the lens covers the area from $k_-(z, x)$ to $k_+(z, x)$, centered on $k_0(z, x)$. These can be shown to be

$$k_+(z, x) = d_b \left[-\frac{x}{z} + \frac{w_l}{2} \left(\frac{1}{z} - \frac{1}{d_o} \right) \right], \quad (5)$$

$$k_0(z, x) = d_b \left(-\frac{x}{z} \right), \quad (6)$$

$$k_-(z, x) = d_b \left[-\frac{x}{z} - \frac{w_l}{2} \left(\frac{1}{z} - \frac{1}{d_o} \right) \right], \quad (7)$$

where Eq. (1) has been used to remove f from Eqs. (5) and (7).

From these equations it is possible to derive a function describing which parts of the screen are illuminated by which illumination bars for any position of the eye. Assume that the illumination system is divided into N equal-width bars of infinite height. Number the bars from 1 to N , left to right. Parameterize the screen width into the range $p \in [0, 1]$, where $p = 0$ represents the left edge and $p = 1$ the right edge of the screen. It can then be shown that the bar B illuminating position p on the screen for a pupil at (z, x) is

$$B(p, z, x) = [b(p, z, x)], \quad (8)$$

$$b(p, z, x) = N \left[\frac{1}{2} + \frac{d_o}{w_o} \left[-\frac{x}{z} + \left(p - \frac{1}{2} \right) w_l \left(\frac{1}{z} - \frac{1}{d_o} \right) \right] \right], \quad (9)$$

where $[a]$ is the nearest integer greater than or equal to a , and $b \leq 0$ or $b > N$ is unilluminated. From this it can be seen that the behavior of a Cambridge

autostereoscopic display is completely specified by the parameters d_o , w_o , w_l , and N .

This result is important because all these parameters are in user space. This permits an autostereo display to be specified without reference to particular optical components, and it provides the designer freedom to use whatever components are necessary to implement the design. It also permits measurement of the parameters of an existing display without the need to know the internal mechanisms of the device.

4. Viewing Zones

The positions of the viewer's eye at which the entire screen appears illuminated determine the useful viewing zone of the display. We can find bounds on this zone by setting $b(0, z, x) = 0$, $b(0, z, x) = N$, $b(1, z, x) = 0$, and $b(1, z, x) = N$. This gives the lines

$$x = \pm \left[\left(1 - \frac{z}{d_o} \right) \frac{w_l}{2} + \left(\frac{z}{d_o} \right) \frac{w_o}{2} \right], \quad (10)$$

$$x = \pm \left[\left(1 - \frac{z}{d_o} \right) \frac{w_l}{2} - \left(\frac{z}{d_o} \right) \frac{w_o}{2} \right]. \quad (11)$$

Figure 5 illustrates the zones defined by these lines within which an image is visible on the screen. Note that there are three distinct zones: an umbra

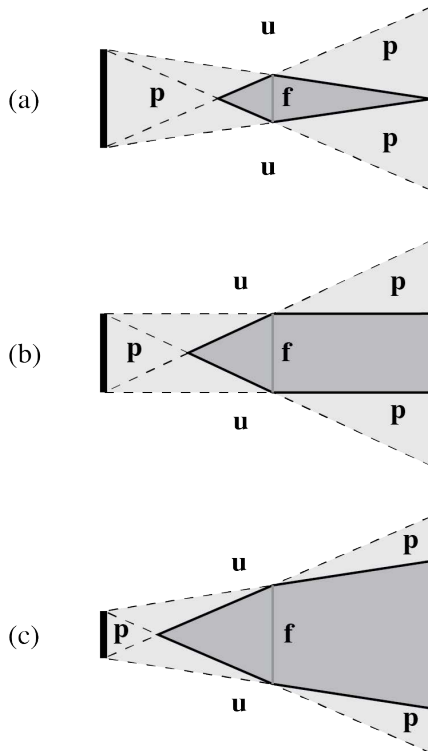


Fig. 5. Three potential configurations of a Cambridge autostereoscopic display. In each the thick vertical bar at the left represents the display's screen, of width w_l , and the thin vertical line is w_o wide at a distance d_o from the screen. (a) Screen wider than viewing zone, (b) screen and viewing zone the same width, (c) viewing zone wider than the screen. Each figure shows the umbra (**u**), penumbra (**p**), and fully illuminated zone (**f**).

(**u**), where nothing is visible on the screen, a penumbra (**p**), where part of the screen is illuminated, and a fully illuminated zone (**f**), where an image is visible across the entire screen. In order to see an autostereoscopic image, both of the observer's eyes must be within the fully illuminated zone.

The equations show that, for a given w_o , a larger screen size w_l leads to a smaller fully illuminated zone. In contrast, for a given screen size w_l , a larger w_o leads to a larger fully illuminated zone. There are, however, limits to the size of w_o for a given number of views.

The maximum useful size of w_o is delimited by the number of views, N , and the human eye separation, s_e . This has an average value of 65 mm for adult males and 63 mm for adult females.¹² If $w_o > Ns_e$, then there will be positions at $z = d_o$ where both eyes see the same view, and hence a monoscopic image is perceived. It is thus necessary to restrict $w_o \leq Ns_e$. Furthermore, for a finite number of views, there will be some value of z beyond which parts of the image will appear monoscopic for the same reason. This value can be shown to be

$$z_{\max} = d_o \frac{Ns_e}{w_o}. \quad (12)$$

This can be considered the farthest distance at which a completely stereoscopic image is visible. However, for cases in which $w_o < w_l$ [e.g., Fig. 5(a)], it is possible that this limiting position could be that at which both eyes can just see an image, in which case

$$z_{\max} = d_o \frac{w_l - s_e}{w_l - w_o}, \quad (13)$$

z_{\max} thus being defined by Eq. (12) for $w_o \geq w_l$ and by the minimum of Eqs. (12) and (13) for $w_o < w_l$. The limiting position close to the screen is that distance at which both eyes can first see an image across the whole screen:

$$z_{\min} = d_o \frac{w_l + s_e}{w_l + w_o}. \quad (14)$$

5. Examples and Comments

To facilitate understanding of these results, consider two examples. Figure 6 shows the current eight-view implementation of the Cambridge display, whereas Fig. 7 shows a proposed 15-view design with a larger screen. These mosaic diagrams show which parts of the screen are illuminated for each eye at a variety of locations. Figure 8 is the key to interpreting each element of the mosaics. Left and right eye images are placed one above the other to show the amount of stereo disparity between the different parts of the image.

These diagrams show that, at any distance other than the optimal, d_o , the image perceived by each eye will contain parts of two or more views. A second

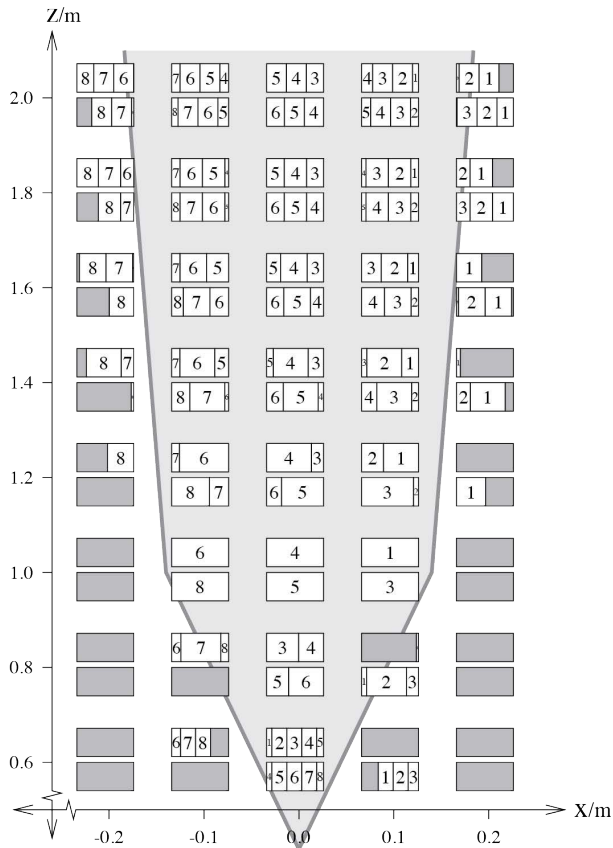


Fig. 6. Mosaic diagram for the current eight-view autostereo display (see Fig. 8 for the key). Parameters are $w_l = 200$ mm, $w_o = 280$ mm, $d_o = 1$ m, and $N = 8$.

consequence is that stereo fusing of the pair of images will contain areas of differing stereo disparity. For example, in Fig. 6 at $z = 1.2$ m, there are regions where the disparity between the eyes is one view and regions where it is two views.

In practice, with eight views, both of these effects tend to be noticeable only when the viewer moves his or her head. Figure 9 shows photographs of the screen of the eight-view display. It can be seen that the interface between views is barely noticeable, because of the similarity between adjacent views. When the head is moved, however, the fact that the image is made up of parts of several views manifests as a wiping effect: the discontinuities move across the screen. The differing stereo disparities in different parts of the picture manifest as a wobbling effect: as the disparity between the two images changes, the perceived depth of objects changes also and they can appear to wobble forward and back. This depth wobble does not occur at or near the optimal distance, nor does it occur for objects at or near the plane of the screen where disparity is zero.

Doubling the number of views, from eight to 16, is observed to improve significantly the three-dimensional illusion during head movement by reducing both of these artifacts. This is because each view is closer in content to its adjacent views than with eight views, reducing the wiping effect. In addition,

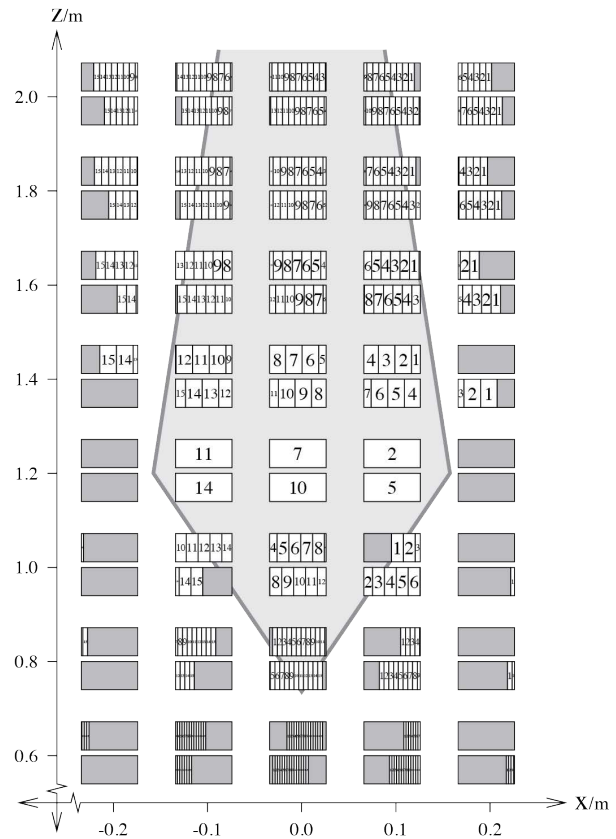


Fig. 7. Mosaic diagram for a proposed 15-view autostereo display (see Fig. 8 for the key). Parameters are $w_l = 500$ mm, $w_o = 315$ mm, $d_o = 1.2$ m, and $N = 15$.

the differences in disparity in a pair of images are also reduced. The limit, as the number of views increases, is to produce a perfectly smooth three-dimensional illusion. In practice even as few as six

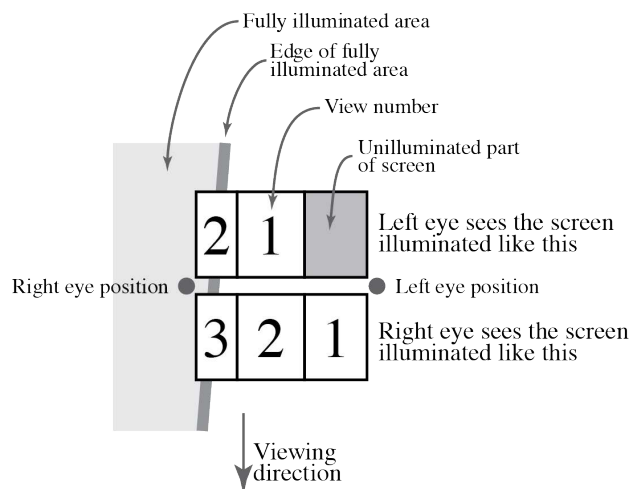


Fig. 8. Key for interpreting the mosaic diagrams in Figs. 6 and 7. Each mosaic diagram contains a grid of these elements. Note that the center of the display's screen is at $(z, x) = (0, 0)$ and that the axes on the mosaic diagrams do *not* meet at this origin.

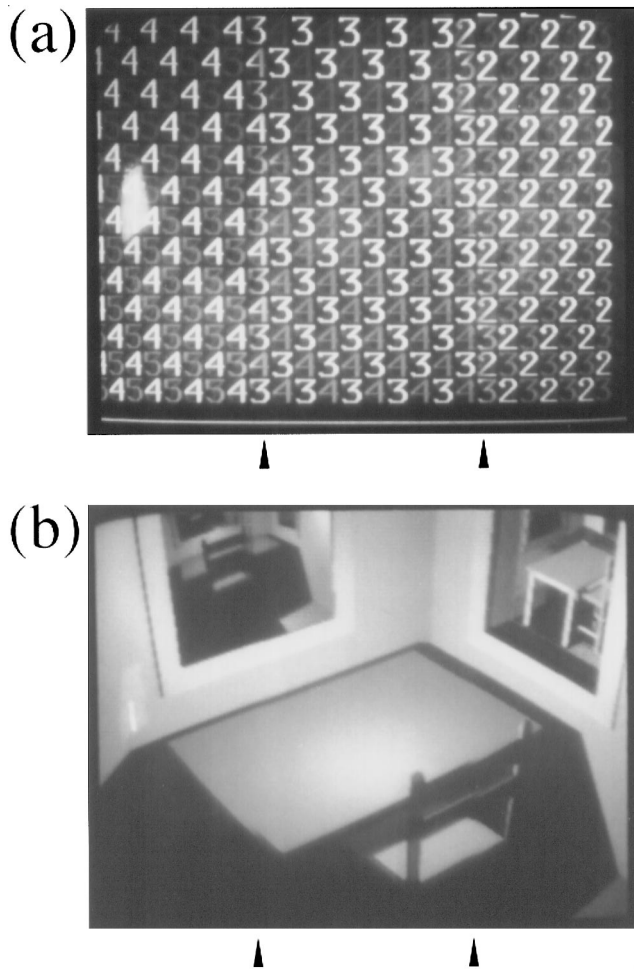


Fig. 9. Photographs from the current display's screen. (a) Test pattern where each view is filled with digits showing the number of that view. This clearly shows the boundaries between views. Views 4, 3, and 2 illuminate different parts of the screen from this position. (b) Same location as (a), but with a computer-generated image of a room. The positions of the boundaries between the views are shown with arrows. The only noticeable artifacts of these boundaries are the slight discontinuities at the left edge of the table (above the left arrow) and at the back of the chair (above the right arrow). This demonstrates that, at least when the head is kept still, the discontinuities produce little degradation in the perceived image.

views produces an acceptable three-dimensional effect for viewers near to the optimal distance.

6. Conclusion

A quantitative description of the behavior of a Cambridge autostereoscopic display has been presented.

The equations derived in this paper allow for the calculation of the viewing zone of a Cambridge display, and for the determination of what a viewer will see from any position in front of the display. It has been shown that these are completely determined by the four parameters, d_o , w_o , w_l , and N [Eq. (9)]. These parameters can be used to specify the design of a Cambridge display, and suitable f , d_b , and w_b can be chosen to implement the design. The equations derived here are therefore useful in both the analysis and design of this type of time-multiplexed autostereoscopic display.

Thanks to E. Murray for typing the original manuscript. This research was funded in part by Autostereo Systems Limited.

References

1. T. Okosi, *Three-Dimensional Imaging Techniques* (Academic, New York, 1976).
2. H. Isono, M. Yasuda, D. Takemori, H. Kanayama, C. Yamada, and K. Chiba, "50-inch autostereoscopic full-color 3-D TV display system," in *Stereoscopic Displays and Applications III*, S. S. Fisher and J. O. Merritt, eds., Proc. SPIE **1669**, 176–185 (1992).
3. J. Hamasaki, M. Okada, and S. Utsunomiya, "Autostereoscopic 3D TV on a CRT," SID Dig. **22**, 844–847 (1991).
4. P. St-Hilaire, S. A. Benton, M. Lucente, and P. M. Hubel, "Color images with the MIT holographic video display," in *Practical Holography VI*, S. A. Benton, ed., Proc. SPIE **1667**, 73–84 (1992).
5. M. Lucente and T. A. Galyean, "Rendering interactive holographic images," Comput. Graph. Ann. Conf. Ser. 387–394 (1995).
6. G. R. Little, S. C. Gustafson, and V. E. Nikolaou, "Multiperspective autostereoscopic display," in *Cockpit Displays*, D. G. Hopper, ed., Proc. SPIE **2219**, 388–394 (1994).
7. A. R. L. Travis, "Autostereoscopic 3-D display," Appl. Opt. **29**, 4341–4342 (1990).
8. A. R. L. Travis and S. R. Lang, "The design and evaluation of a CRT-based autostereoscopic 3-D display," Proc. Soc. Inf. Disp. **32**, 279–283 (1991).
9. S. R. Lang, A. R. L. Travis, O. M. Castle, and J. R. Moore, "A 2nd generation autostereoscopic 3-D display," in *Seventh Eurographics Workshop on Graphics Hardware*, P. F. Lister, ed., Tech. Rep. EG92 HW (Eurographics, Oxford, U.K., 1992), pp. 53–63.
10. J. R. Moore, A. R. L. Travis, S. R. Lang, and O. M. Castle, "The implementation of a multi-view autostereoscopic display," in *IEE Colloquium on Stereoscopic Television* (Institution of Electrical Engineers, London, 1992), pp. 4/1–4/16.
11. A. R. L. Travis, S. R. Lang, J. R. Moore, and N. A. Dodgson, "Time-manipulated three-dimensional video," Soc. Inf. Disp. Dig. **26**, 851–854 (1995).
12. L. Lipton, *Foundations of the Stereoscopic Cinema* (Van Nostrand Reinhold, New York, 1982).