Balancing the expected and the surprising in geometric patterns

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Abstract

I investigate the trade-off between the expected and the surprising in certain geometric patterns. This work is inspired by Bridget Riley's early Op art pieces, *White Discs 2* (1964) and *Fragment 6/9* (1965). I analyse these two works, investigate a range of variants, and propose hypotheses about the perceptual effects in patterns like these. The key hypothesis is that there is an æsthetically interesting range where between a quarter and a half of a regular pattern is adjusted in some way. I report on a perceptual experiment that tests and supports this hypothesis, and discuss the implications.

Key words: pattern, Op art, æsthetics, design

1. Introduction

Bridget Riley's *Fragment 6/9* (Figure 1) stimulated an informal investigation into the æsthetic trade-off between the expected and the surprising in art of this type [1]. That investigation considered whether there is a correct balance between these two factors that produces the most æsthetically pleasing compositions. This paper builds on that informal investigation, summarising the findings from the earlier paper, reporting on a formal experiment that supports one of the hypotheses proposed there, and discussing the wider context.

2. Background

The twentieth century saw a tremendous range of artistic movements: fauvism, cubism, art nouveau, art deco, Op art, pop art, modernism and post-modernism amongst others. Since the middle of that century the variety of different movements has exploded. Livingstone [2] attributes this to several factors: artists reacting against their immediate predecessors, artists forming groups with colleagues and exchanging ideas, artists being encouraged to try new things by critics, dealers, and museum administrators who depend on new art, and artists being anxious not to be left behind. These combined to create an atmosphere conducive to experiment remote from the taste of the public at large: artists acting as researchers into the limits of their media and into the depths of the human psyche. Many of these experiments have remained remote from the public's taste but some have captured the public imagination. Bridget Riley's early Op art is an example of the latter.

Riley studied art at Goldsmiths College and the Royal College of Art in London in the 1950s. She started investigating Op art in 1960. Her output from 1961 to 1966 consists of black-and-white geometric patterns and variations in shades of grey. From 1967 onwards she used colour, but always her work is restricted to a simple vocabulary of abstract shapes: squares, circles, ovals, lines, stripes, curves [3].

Op art, in general, consists of simple shapes in precise geometric relationships. Most examples are purely deterministic (e.g., Figure 3). The geometry is described by the artist and the work is simply an implementation of that geometry. It is straightforward to write computer programs to emulate many Op art pieces, as demonstrated in Figures 1–3.

Much of Op art's effect depends on phenomena that have been extensively studied by scientists [4]; Livingstone goes so far as to call the movement "quasi-scientific" [2]. Riley is famous for her works that play on the vagaries of human perception [3]. Compare the Riley-inspired composition in Figure 3 (right) with a straightforward chequerboard (Figure 3 left). The plain chequerboard is visually unexciting: there is no surprise to it. This does not mean that it has no artistic value: Carl Andre, in *Aluminium-Copper Alloy Square* (1969),

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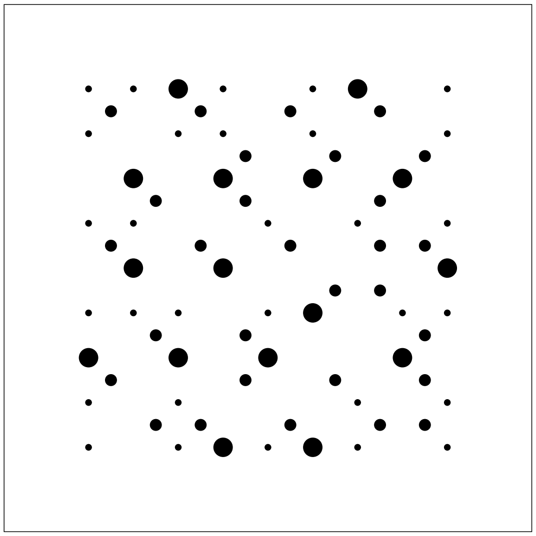

Figure 1: A computer rendition based on *Fragment 6/9*, created using a short PostScript program. When enlarged to the dimensions of the original, the discs are within a millimetre of the size and within a few millimetres of the locations that they have in the original. The measurements were taken from one of the original prints. *Fragment 6/9* is part of Bridget Riley's *Fragments* series, 1965, screen-printed on plexiglass, 625 × 720 mm, limited edition of 75 prints.

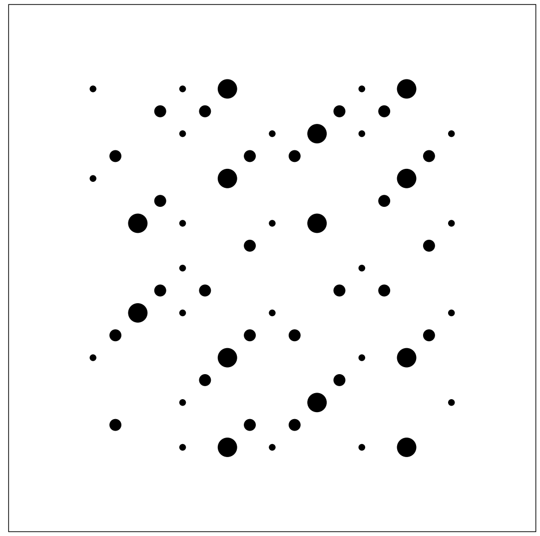

Figure 2: A computer rendition based on *White Discs 2*, created using the same PostScript program as Figure 1. The discs have been assumed to have the same relative sizes and relative spacing as in *Fragment 6/9*. The arrangement of the discs was determined from a reproduction of the original. The original, by Bridget Riley, was created in 1964, emulsion on hardboard, 104 × 99 mm.

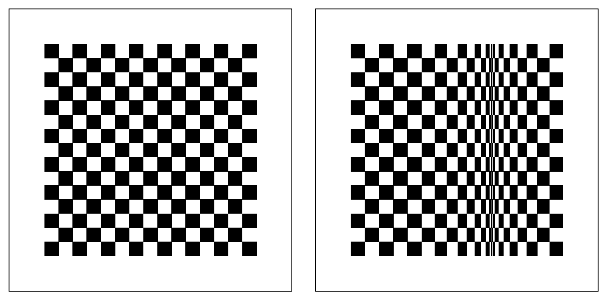

Figure 3: Two computer renditions. At left. A regular chequerboard. At right, a modified chequerboard inspired by Riley's *Movement in Squares* (1961). ©2008, Neil A. Dodgson, used with permission. (see note 1)

deliberately used the unsurprising nature of the chequerboard pattern to deflect attention from the artwork and onto the space in which it was installed [2]. Riley's adaptation of the chequerboard (Figure 3 right), by contrast, adds an element of surprise. This may well be owing to the visual system constructing a 3D representation and the conscious brain then trying to reconcile an obviously flat construction of black and white rectangles with its perception of a surface strongly curved in depth [4]. Many of Riley's works play on similar effects.

By contrast, the two works considered in my previous paper [1], *White Discs 2* (1964, Figure 2) and *Fragment 6/9* (1965, Figure 1), cause no depth effects and limited visual incongruity. They are less well known than Riley's visually disturbing works. Nevertheless, I hypothesised that they interact in an æsthetically pleasing way with the pattern detection mechanism in the human brain; their æsthetic appeal arising from a good balance between the expected and the surprising.

In my earlier paper [1], I suggested that the æsthetic optimum was somehow related to nature, in which we see a similar balance between the expected (a tree is a tree) and the surprising (every tree is different): we somehow have the ability to filter out the differences and perceive the underlying type or pattern. This theory, that art is beautiful to the extent that it imitates nature, is an old one. It has several inherent problems [5], not least being that it does not explain much contemporary art, but it is somewhat supported by the theory that æsthetic appreciation is a by-product of evolutionary selection, which was driven by a need to survive within the natural world and hence to perceive that world efficiently and effectively [6]. Indeed, the neurobiologist Zeki holds the view that "...artists are neurologists, studying the brain with techniques that are unique to them and reaching interesting but unspecified conclusions about the organisation of the brain" [7]. Mondrian expressed a similar view that the ultimate goal of abstract art is the expression of pure reality: "Despite cultural lags and breaks, there exists a continuous progression in the disclosure of true reality by means of the abstraction of reality's appearance" [8]. Riley has said "I draw from nature. I work with nature although in completely new terms" [9], though she is also clear that she produces her work by trial and error, not through scientific method.

Different commentators analyse Riley's pieces in different ways. Kudielka [10] asserts that *White Discs 2* is part of Riley's preparation to move into colour, investigating "new forms of field organisation." He continues "Riley created a powerfully elusive sensation by deleting parts of the regular structure spanning the field. Once the 'white discs' have started to emerge under one's eyes, it is not difficult to intimate the complete field of black discs." This reading of the work supports the analysis in my earlier paper [1]. Other commentators have a different take on the work, which makes the regular underlying pattern peripheral to the principal perceptual effect. Searle [11] and Moorhouse [12] understand the work in terms of the strong after images (the white discs) that appear on prolonged observation. Moorhouse analyses the effect into three stages: initially the eye simply sees rows of black discs of varying sizes but this is quickly overtaken by a stage which is marked by the appearance of numerous ghostly white discs—the after images of the black discs. The viewer then moves into a steady state in which there are the static black discs and the dynamic white discs, which move, appear, and disappear as the eye roves over the image. This effect is much more pronounced in the large-scale originals than in the small versions printed here. Moorhouse's interpretation is supported (over Kudielka's) by consideration of *Fragment 6/9*, where the underlying pattern is perturbed mostly by moving discs away from their places within a regular pattern rather than by deleting discs, and by consideration of *White Discs 1* [13, p. 30] which is more akin to *Fragment 6/9* than to *White Discs 2*.

Nevertheless, our interest is in these works as perturbed patterns rather than in the perceptual illusions they cause. The question being why Riley chose these particular arrangements of discs as being somehow the "right" arrangements (c.f., Gombrich's discussion of "right" [14, pp. 32, 583]). Consideration of these two works as perturbed patterns led to informal experiments and then to hypotheses about the human reaction to patterns of this general type [1]. These hypotheses can be stated, briefly, as follows:

1. Humans can distinguish finely between different types of pattern.
2. To produce an æsthetically pleasing effect, modification away from a regular pattern cannot be purely random: some notion of balance must be used.
3. Humans can easily detect patterns when less than 25% of the pattern is perturbed, whereas perturbation of over 50% of the pattern destroys it, and there is therefore an æsthetically interesting range between 25% and 50% perturbation.

Before considering these hypotheses in detail, I explain the scientific context for this exploration of æsthethic balance and then summarise my previous analysis of Riley's two works. In the sections that follow, I discuss each of the hypotheses in more detail including details of a psychophysical experiment whose results support the third hypothesis.

3. The Scientific Context

There is a long history of perceptual experiment on simple stimuli [4] and of attempts to reduce æsthetics to science [6]. The field is so large that, even in the 1950s, Arnheim was compelled to write "…it is probably beyond the power of any one person to give a fully satisfactory survey of the relations between the theory of the visual arts and the pertinent work in psychology" [15, p. 7].

There is, of course, considerable debate about the validity of applying scientific method to æsthetic response [16, 17]. For example, Birkhoff undertook early work on reducing æsthetics to mathematics. He started by studying polygons "a class of æsthetic objects of the utmost simplicity" [18]. While his work has been described as "misguided" [19], his motivation remains relevant: "The true function of the concept of æsthetic measure is to provide systematic means of analysis in simple formal æsthetic domains. There is a vast difference …between the creation of a work of art and an analysis of the formal factors which enter into it" [18, Preface, p. ix]. The contrast between this laudable motivation and the limitations of his approach serves as a warning not to overstate the importance of one's results.

More successful was the empirical, rather than mathematical, Gestalt theory. The foundations of our knowledge of visual perception were laid in the laboratories of the Gestalt psychologists [15]. Gestalt psychology was founded by Max Wertheimer, Wolfgang Köhler [20] and Kurt Koffka [21]. The name derives from earlier work by Christian von Ehrenfels: "Über Gestaltqualitäten", translated literally as "On Form-Quality." The change in understanding from the prevailing nineteenth century view to Ehrenfels to Wertheimer can be simplified to three stages: "First, the associationist notion was that the whole is *equal* to the sum of its parts. With Ehrenfels' formulation, it was believed that the whole is *more* than the sum of its parts—the whole equals the sum of its parts *plus* another element, the Gestalt quality. But for Wertheimer, the new Gestalt theory was founded on the position that the whole is entirely *different* from the sum of the parts, indeed is *prior to* the parts; wholes are integrated, segregated systems that have an inherent structure of their own, and the structure of the whole in fact determines the nature of the parts." [22].

Gestalt theorists produced a range of "laws" of how stimuli generate perceived structures: principles such as proximity, similarity, continuity, closure, and symmetry [23, Ch. 6]. They generally presented their ideas through examples: if the reader looked at a stimulus and perceived a visual effect, then that demonstrated the writer's theory [21, p. 64]. This empirical approach is considered a weakness of the theory by its critics: it describes the perceptual effects but does not explain them [23, p. 110]. Marr maintains that the Gestaltists' problem was that they were ignorant of mathematics; that, in order to understand human perception, one needs to be familiar with psychology, physiology, mathematics, and computation [24, p. 187].

The basic law of visual perception, for the Gestaltist, is that of Prägnanz: any stimulus pattern tends to be seen in such a way that the perceived structure is as simple as the given conditions permit [15, p .53]. That is: when there is more than one possible organisation that can be perceived, we will see the simplest possible [25]. But simplicity is not enough: if simplicity were the one overriding goal of art, evenly stained canvases or perfect cubes would be the most desirable art objects [15, Ch. IX]. Indeed some artists have presented us with evenly stained canvases (e.g., pure white from Robert Ryman, *Twin* (1966) and pure black from Ad Reinhardt, *Abstract Painting* (1963) [26]). But even the most dedicated colour field artists (e.g., Rothko, Still, Newman) generally present more to the eye than an evenly stained field.

Many researchers, including Arnheim [15] and Marr [24], have undertaken considerable fruitful research in an attempt to explain the mechanisms that underlie the Gestaltists' and others' observations of human perception. Hochberg, for example, tackled the problem by attempting an information theoretic approach to figural "goodness" [27]. This work, like Birkhoff's before it, only applied to a very limited set of stimuli [25]. He warns: "As long as the perception psychologist finds it necessary to employ only constrained and simplified pictures in his research, it would be very unwise to employ his generalisations with more confidence than these constraints display." Thus, whatever we may learn from analysing Bridget Riley's works and derivatives thereof, we have still learnt only about the human perceptual response to a specific type of simple stimulus.

4. Analysis of *White Discs 2* and *Fragment 6/9*

Consider both *Fragment 6/9* (Figure 1) and *White Discs 2* (Figure 2). At first sight, these may appear little more than a random jumble of variously sized black discs on a white background. In both cases, however, there is an underlying regular pattern. There is a balance between the expected and the surprising.

To move on to an informal investigation of this balance, it is necessary to find an underlying order in Riley's compositions. It is easy to see that the discs come in just three sizes (call them large, medium, and small) and that they are located at the corners of a regular diamond-shaped grid (this grid is most obvious in Figure 8 (A/E)). It is less obvious that, if we consider the work as a set of horizontal rows of discs, the medium discs appear exclusively on every even row, with the odd rows consisting exclusively of large and small discs. From this starting point, consider the two works individually.

4.1. White Discs 2

It seems clear that the pattern in *White Discs 2* (Figure 2) is generated by removing discs from a particular regular pattern (Figure 8 C/G). This interpretation is supported by Riley's sketches for the work [28]. About 30% of the discs are removed from the regular pattern to create the artwork. I consider the significance of this proportion in Sections 7 and 8.

4.2. Fragment 6/9

Fragment 6/9 is more challenging. The discs' sizes and their adherence to a grid demonstrate that it is related to *White Discs 2* but there is insufficient evidence in the artwork to allow us to be certain of recovering Riley's exact underlying regular structure, assuming that one ever existed.

In my previous paper [1], I proposed Figure 4 (left) as the regular underlying structure. To get from this to the artwork requires that 33% of the discs be perturbed, i.e., moved (24%), deleted (6%), or inserted (3%). I took this 33% to be indicative of the required proportion of surprise. However, Figure 4 (left) is not totally regular: the top and bottom rows of discs are special. This is because the top and bottom rows of the artwork itself (Figure 1) appear to be special and therefore I constructed

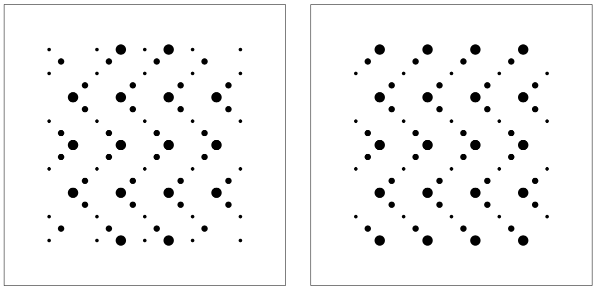

Figure 4: Two regular patterns that might underlie Fragment 6/9. At left, the pattern proposed in my earlier paper [1]. At right, a more regular version. ©2003, 2008, Neil A. Dodgson, used with permission.

this "regular" version so that its top and bottom rows are each an exact copy of the bottom row of the artwork. If one insists on a more regular underlying pattern, then I propose Figure 4 (right). To get from here to the artwork requires that 43% of the discs be perturbed, i.e., moved (20%), deleted (9%), inserted (12%), or changed in size (2%). Both 33% and 43% are within the experimentally determined bounds discussed below (Sections 7 and 8).

5. Hypothesis 1: Distinguishing Types

The first hypothesis is that humans have considerable ability to distinguish finely between types or "species" of pattern. For example, Figure 5 shows patterns generated by three algorithms. You should be able to distinguish which pair is generated by each algorithm. One algorithm places discs at purely random locations within a square; the second places them randomly restricted by the constraints identified in the second paragraph of Section 4; the third starts with the regular arrangement shown in Figure 4 (left) and manipulates one third of the discs in roughly the proportions identified in Section 4.2.

This hypothesis would be straightforward to test. A variant of Wallraven, Cunningham and Fleming's method [29] could be effective here. However, it is not clear whether we would learn anything new. It is already clear that most humans possess this ability to distinguish between types. It would be possible to develop a test of just how fine a distinction it is possible to make, but it is also clear that humans can be taught to make various degrees of distinction. Consider the identification of tree species. Mitchell says: "As beginners, we all puzzle over the differences between Lawson Cypress and Thuja even with foliage in the hand, and the existence of Nootka Cypress seems to complicate matters beyond solution. Some years later we will unhesitatingly identify all three, and more besides, at half a mile's distance from a passing train." [30, p. 31].

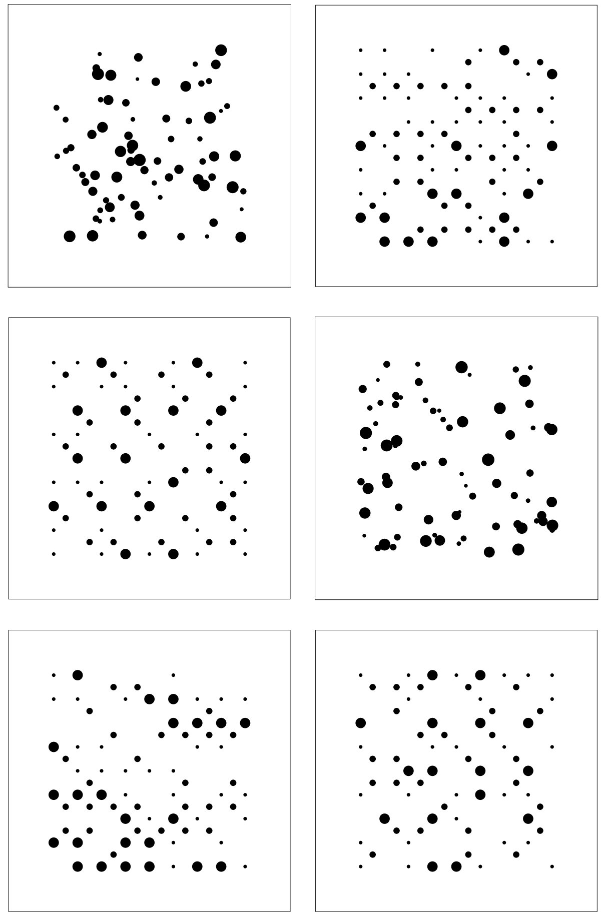

Figure 5: Six variants generated by three algorithms, ©2003, 2008, Neil A. Dodgson, used with permission.

6. Hypothesis 2: Æsthetic Balance

To produce an æsthetically pleasing effect, the hypothesis is that modification away from a regular pattern cannot be purely random: some notion of balance must be used. This militates against the unsophisticated use of a random number generator: true randomness is æsthetically unappealing. For example, the choice of which 30% of the discs to remove from Figure 8 (C/G) has an effect on the æsthetic appeal of the result. Figure 6 shows four such removals. There is clearly a difference in the balance of the four compositions. The hypothesis is that this notion of balance affects the æsthetic response.

It is well documented that balance is necessary [14]. Arnheim devotes a chapter to the concept [15, Ch. 1].

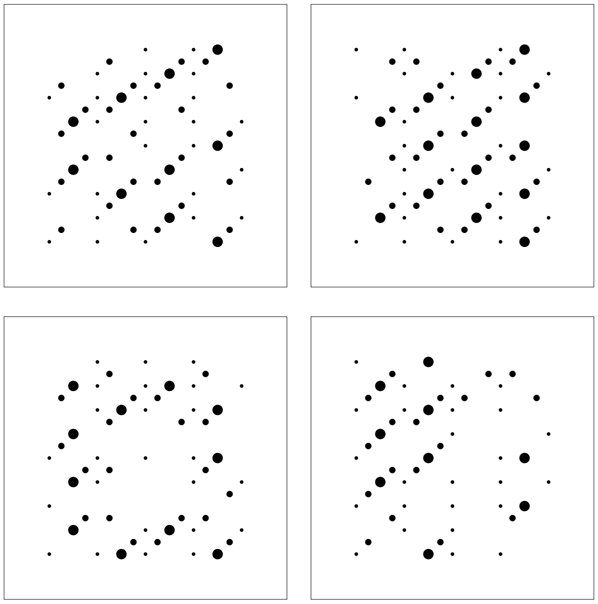

Figure 6: Four examples showing removal of around 30% of the discs from the regular pattern in Figure 8 (C/G). Compare these with *White Discs 2* (Figure 2). All are generated by a short PostScript program, using PostScript's inbuilt pseudo-random number generator with different seeds. The top two are stimuli randomly chosen for use in the experiment described in Section 8. The bottom two were selected as the most unbalanced of 100 pseudo-randomly generated examples; they demonstrate the types of perceptual imbalance that can result from using a random number generator. ©2008, Neil A. Dodgson, used with permission.

Consider Alberti's definition of beauty as a reasoned harmony of all the parts within a body such that nothing can be added, taken away or changed but for the worse [31, 32, 33] (see note 2). The reasoning that led to the second hypothesis is that, in *White Discs 2* and *Fragment 6/9*, Riley has produced pieces which exhibit that reasoned harmony: to add, remove or move a single disc would be for the worse.

The problem facing both the experimental psychologist and the computer programmer is this: can we mathematically codify what is meant by this concept of perceptual balance? Arnheim offers the opinion that "Except for the most regular shapes, no known method of rational calculation can replace the eye's intuitive sense of balance" [15, p. 19]. All the elements in a composition are under directed tension and this must be in perceptual balance for the work to be "right". Indeed, the abstract artist, with her deliberately imposed restrictions on her composition, may find this more difficult than the figure painter [14, p. 583].

Figure 6 demonstrates that it is possible for a random process to get balance wrong. There is much evidence that human beings have poor intuition of "random". For example, the numbers drawn in a lottery are completely random but many people assume determinism when choosing numbers. Metz reports that, although understanding of randomness develops with age, adults still have considerable difficulty with the concept [34]. One of Damien Hirst's assistants tells us that the colours in Hirst's spot paintings are chosen "randomly" (see note 3); observation of the works indicates that the colours are not truly randomly placed, because the occurrence of neighbouring near-identical colours is far less than is generated automatically by a pseudo-random number generator. The human understanding of "random" differs from the mathematical. One suggestion is that the distribution of similar colours should ideally follow a Poisson disc distribution. This requires further research. As another example, Ellsworth Kelly's work *Spectrum Colors Arranged by Chance II* (1951) [26] has colours that were not arranged purely at random, but by more complex mathematical methods [35], producing a result that is visually pleasing.

7. Hypothesis 3: Pattern Perception

The hypothesis is that humans can easily detect patterns when less than 25% of the pattern is removed or disturbed, whereas removal of over 50% of the pattern destroys it. For example, Figure 7 demonstrates the removal of a proportion of the discs ranging from removal of 70% of the discs to removal of no discs. Informal observation of this and other patterns led me to the hypothesis that removal of 50% or more of the discs leaves insufficient of the pattern for the brain to spot any underlying structure to the disc's arrangement. At the other end of the spectrum, removal of less than about 25% of the pattern leaves a "pattern with holes": the pattern detection in the human brain is able to complete the pattern easily and we see simply an incomplete version of the whole pattern. This ties back to the Gestalt theory of Prägnanz: if the simplest perceptual explanation is a pattern with a few pieces perturbed, then that is what we will see. The removal of between about 25% and about 50% of the discs produces a result which has sufficient structure for the underlying pattern to be discernible and sufficient lack of structure for effort to be required to discern that underlying pattern. The work is thus seen as a work in its own right, rather than an imperfect version of the pattern. The trade-off between the expected and the surprising operates, I believe, in two opposing, complementary, ways: we expect a pattern and are surprised by the deviation from the pattern; and, conversely, we

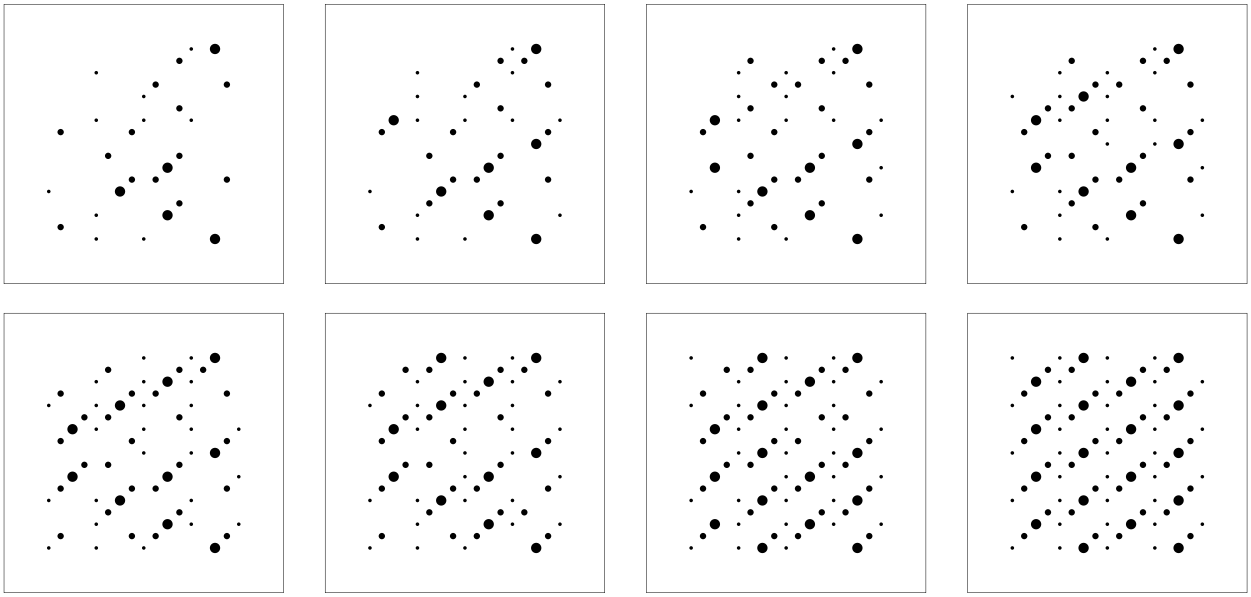

Figure 7: Eight examples showing (top row) 30%, 40%, 50%, 60%, (bottom row) 70%, 80%, 90%, 100% of the discs from the regular pattern in the bottom right, ©2008, Neil A. Dodgson, used with permission.

expect no pattern and are surprised by the hints of pattern that emerge on prolonged viewing. This hypothesis suggests that there is something important about Riley's artistic decision to remove around one-third of the discs in *White Discs 2* and that there is an æsthetically interesting range between 25% and 50% removal. This hypothesis is examined in the experiment in the next section.

8. Experimental Investigation

I conducted a perceptual experiment to test the third hypothesis. The experiment refines the hypothesis to the two questions: How much of a pattern needs to be present for it to be immediately obvious to a human observer? How much of a pattern can be removed before it ceases to be obvious to a human observer?

The informal results suggest that the first number is around 75% and the second around 50%. The experimental aim is to confirm or counter the informal results. These values may be correct, or it may be that the true values are different to the informal results, or it may be that the two values are not significantly different, which would imply that there is no "æsthetically interesting range" in which a pattern is not obvious but may still be discerned.

8.1. Experimental design

Participants were recruited from amongst the graduate students and staff of my department. Participants were presented with 32 stimuli. The 32 stimuli were divided into eight trials, labelled A–H, of four runs each. Each stimulus was a pattern of discs, where discs either appear or disappear in controlled steps. Four different patterns (Figure 8) were presented in two complementary ways:

Increasing (trials A–D): the screen started blank. Every 1.5s a randomly selected 5% of the pattern would be added, until, after 30s, the entire pattern was visible. Participants were asked to indicate at what point the pattern became obvious to them.

Decreasing (trials E–H): the stimulus started with the entire pattern visible. Every 1.5s a randomly selected 5% of the pattern would be removed until, after 30s, the screen was blank. Participants were asked to indicate at which point the pattern ceased to be obvious. In the analysis I assume that the pattern was still obvious on the step immediately before the one indicated, that is, five percentage points higher.

The four patterns (Figure 8) were: a regular pattern with three disc sizes and many lines of symmetry (A/E), a sunflower pattern (B/F) [36, 37], the regular pattern

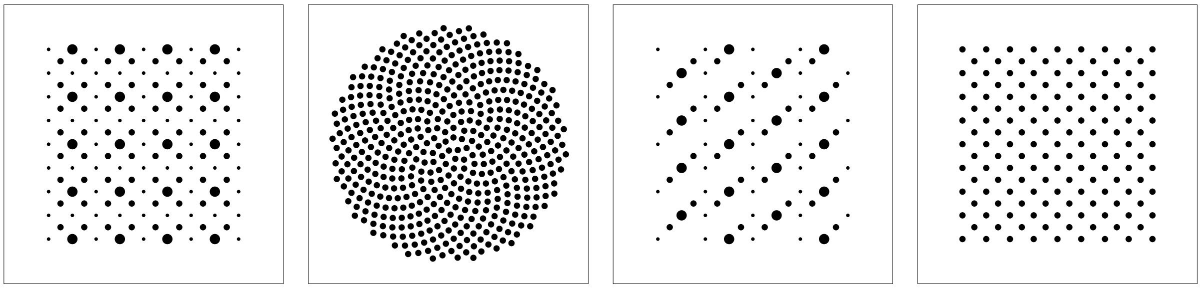

Figure 8: The four stimuli used in the experiment, ©2003, 2008, Neil A. Dodgson, used with permission.

underlying *White Discs 2* (C/G), and a regular pattern with a single disc size (D/H).

In each trial (A–H) the same pattern was presented four times (the four "runs"). The randomness of the removal or introduction of pattern elements was different in each of the four runs within a trial, but was the same for all participants and the same in corresponding runs of the insertion and deletion tests. The latter constraint allows comparison of results from identical stimuli in the two types of tests.

8.2. Experimental results

Thirteen participants took part. Of these, one self-reported that he had considerable prior experience of pattern identification and suspected that his results would be atypical. Inspection of the results verified this. In particular, this participant had the minimum score of all participants on the first 22 of the 32 trials. His data was discounted in the subsequent analysis, which is therefore based on twelve participants.

On the first increasing trial for each pattern (A–D), participants were unaware of the final pattern. On the remaining three runs for a pattern, participants knew what pattern to expect. This led to a suspicion that the first run would be statistically different to the other three for a given stimulus. Paired t-tests indicate only two runs had statistically significant differences to the other three runs for that stimulus, at $p < 0.05$. These were the first run of stimulus D (as suspected) and the *fourth* run of stimulus C (not as suspected). Even these are not significant at $p < 0.01$. I therefore use data from all runs in the analysis.

Figure 9 (top) plots the results from the increasing (A–D) and decreasing (E–H) trials. Figure 9 (middle, bottom) shows the histograms individually, each column broken down by stimulus. This shows that only stimuli B (sunflower) and C (underlying pattern for *White Discs 2*) contribute above 85%. Figure 9 (middle, bottom) also include normal distributions, with the same mean and standard deviation as the associated histogram. For trials A–D, mean is 64.3%, standard deviation is 14.3; for trials E–H, mean is 52.5%, standard deviation is 10.6.

The data is broken down into quartiles in the box plot of Figure 10 (top). This shows minimum, lower quartile, median, upper quartile, and maximum for each stimulus. The data for stimuli E–H have had five percentage points added to show the last data point for which the pattern is obvious, in order that they can be compared with the results for stimuli A–D.

Finally, because the stimuli for corresponding increasing and decreasing runs are identical, we can plot a histogram (Figure 10 (middle)) of the difference between the first step at which a pattern becomes obvious, in the increasing run, and the last step at which a pattern remains obvious, in the decreasing run. The mean is 16.8 percentage points, with standard deviation of 13.0.

8.3. Discussion of Experimental Results

It appears clear from the graphs that there is a significant difference between the point at which a pattern becomes obvious in the increasing runs and the point at which a pattern ceases to be obvious in the decreasing runs. This is confirmed by a paired t-test between the increasing runs and the corresponding decreasing runs (adjusted up by five percentage points), which is significant at $p < 0.001$. This indicates that there *is* a range of values for which the pattern is not obvious but for which a pattern may still be discerned.

The box plots (Figure 10 (top)) demonstrate variance amongst the different stimuli. However, only one pair of patterns has statistically significant differences ($p < 0.05$) on both the increasing and decreasing stimuli (C/G against D/H). The other five possible pairs do not have a statistically significant result on this test. This indicates that the different patterns are not causing statistically significantly different results, and it is therefore reasonable to consider all the results together.

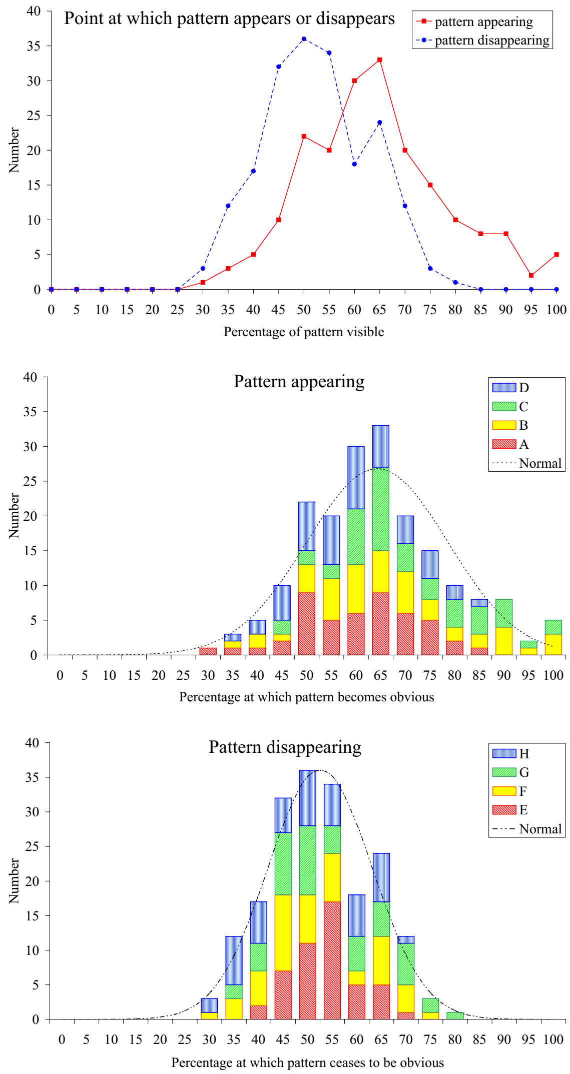

Figure 9: Experimental data. *Pattern appearing*: the point at which the participants said that the pattern became obvious to them, as an increasing amount of the pattern was presented. *Pattern disappearing*: the point at which the participants said that the pattern ceased to be obvious to them, as a decreasing amount of the pattern was presented.

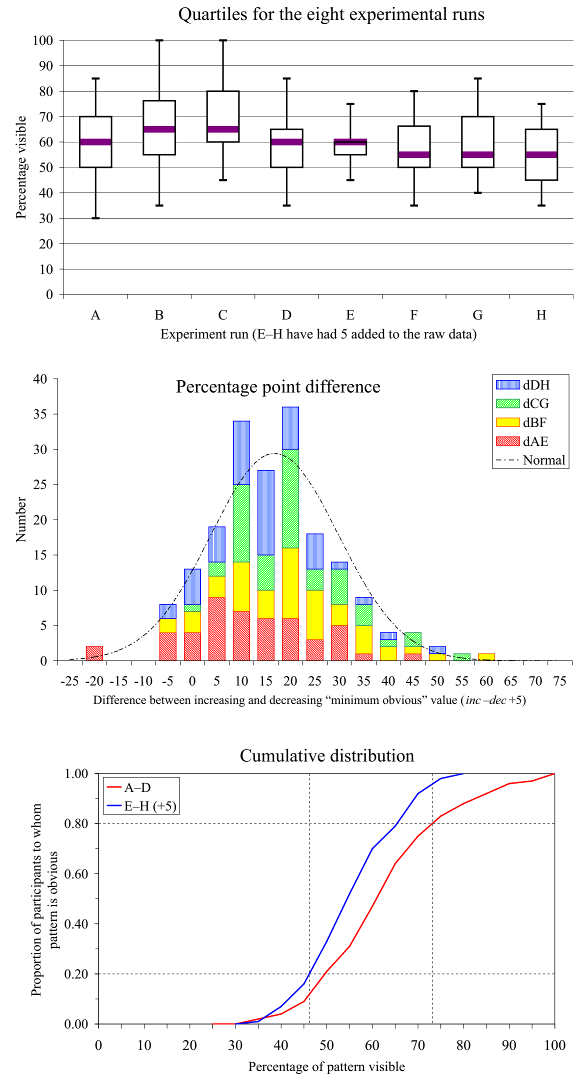

Figure 10: Quartiles, percentage point difference, and cumulative distribution. In all three cases, the data for "pattern disappearing" has been incremented by five percentage points to the stimulus that was immediately before the one on which the participant said that the pattern ceased to be obvious, i.e., to the last stimulus in which the pattern was still obvious.

To support the hypothesis, we need to know the point above which it is likely that a pattern will be discerned and the point below which it is unlikely to be able to discern the underlying pattern. The cumulative distribution in Figure 10 (bottom) shows that, on the increasing stimuli, the pattern is obvious in over 80% of cases where 73% of the pattern is visible, and that, on the decreasing stimuli, the pattern is not obvious in 80% of cases where less than 46% of the pattern is visible. If the pattern is not known, as is the case if the perturbed pattern is presented as an artwork, then we need to consider only the increasing stimuli. This increases the lower value from 46% to 50%. This supports the hypothesis that the æsthetically interesting range is between about 50% and about 75% of the pattern being present.

There are two interesting anomalies. One is the small number of results at 100% indicating that the pattern did not become obvious to the participant until the entire pattern was visible, with a further 10 observations at 90% or 95%. All of these are attributed to stimuli B and C. For B, seven of the eight high values come from just two participants. For C, they come from five participants.

The other anomaly is in the difference chart (Figure 10 (middle)) where ten results indicate that, on exactly the same stimulus, the lowest point at which the pattern remains obvious on the decreasing run is above the point at which it becomes obvious on the increasing run. This indicates the difficulty of making a precise judgements of "obvious."

The experimental results provide some support for the third hypothesis. They demonstrate that there is a range in which the pattern is not *immediately* recognised but in which the pattern *can be* recognised given some effort. This results in a new hypothesis that the æsthetic interest lies in forcing the brain to do some work. The æsthetic balance lies in providing sufficient information that the viewer can reconstruct the pattern but not so much information that the pattern is obvious.

The link from this result to Riley's art work is simply that Riley's works do lie in this region where a pattern is hinted at but not made obvious. The new hypothesis implies that it is the brain's natural search for regularity, simplicity and pattern that make the work stimulating. Other artist's work could be addressed in a similar manner. Ellsworth Kelly and Damien Hirst have already been mentioned. A search through just one book of abstract art [38] provides a range of other works that may be usefully investigated (see note 4).

9. Conclusions

The patterns analysed, created, and used as experimental stimuli are all regular patterns that have been perturbed in some way to create an æsthetic effect. They are perceptually simple and the underlying assumption is that their simplicity makes them a useful testbed for experimenting with notions of æsthetic balance.

The particular hypothesis that is experimentally tested here concerns the ability of humans to detect an underlying pattern, when a certain proportion of its elements have been disturbed. There are four parts to the hypothesis as originally stated [1]: (i) that humans can easily detect patterns when up to about 25% of the pattern is removed or disturbed, (ii) that removal of over about 50% of the pattern destroys it, (iii) that there is an æsthetically interesting range between these two values, and (iv) that a good, artistic, balance is achieved by retaining about two-thirds of the pattern, while manipulating the other one-third in some way. The experimental results support the percentages in (i) and (ii). Parts (iii) and (iv) are less amenable to mathematical analysis but we can tentatively conclude that the evidence supports them also.

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Notes

1. All images used in this paper are derived works generated by computer programs. All images in this paper remain in the copyright of their creators, as noted in the figure captions, and are reproduced with permission.

2. Alberti uses the concept of beauty throughout *Della pitura* ("On painting") without ever defining it [32]. The definition of beauty quoted in Section 6 comes from *De re aedificatoria* ("On the art of building") [31], VI, ii: "ut sit pulchritudo quidem certa cum ratione concinnitas universarum partium in eo cuius sint: ita ut addi / aut diminui / aut immutari possit nihil quin improbabilius reddatur."

3. The reference to randomness in Hirst's spot paintings is by personal communication, via Prof. Peter Robinson. To be fair, it is likely that Hirst's assistant was using "random" to mean a particular type of æsthetic balance rather than to mean any particular mathematical notion. This supports rather than detracts from the comments made in Section 6.

4. The works in Optic Nerve [38], alluded to in Section 8.3, that might be most amenable to similar analysis are those of Lohse, Fig. 44; Benjamin, 46, 139; Josef Albers, 45; Poons, 73, 143; Davis, 154; Vaserely, 155; and Anni Albers, 238.

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